$$n=\tfrac{1}{2}[(8+3\sqrt{7})^r+(8-3\sqrt{7})^r] \text{ and } m=\frac{1}{2\sqrt{7}}[(8+3\sqrt{7})^r-(8-3\sqrt{7})^r],$$

where for r successive values 1, 2, 3, ... may be put. Since $y=1$, $x=4$ satisfy equation $y^2-7x^2=-111$, we have $q=1$, $p=4$, and thus we find $y=n\pm28m$, $x=m\pm4n$. Substituting for r the numbers 2, 3, 4, we get the sets

$$\begin{array}{llllll} y=76, & y=92, & y=1217, & y=1471, & y=309119, & y=373633, \\ x=29, & x=35, & x=460, & x=556, & x=116836, & x=141220, \quad \text{etc.} \end{array}$$

Thus $y=1471$ is the least prime which satisfies the equation.

Also solved by A. H. Bell.

GEOMETRY.

290. Proposed by G. W. GREENWOOD, M. A., McKendree College, Lebanon. Ill.

Show that the point (1, 1) is a conjugate point on the locus $x^3+y^3-3xy+1=0$.

I. Solution by the PROPOSER.

If a line through the point (1, 1) making an angle θ with Ox have a point P in common with the locus, the coördinates of P, *i. e.*, $1+r\cos\theta$, $1+r\sin\theta$, where r is the distance of P from the point (1, 1), satisfy its equation. Therefore

$$(1+r\cos\theta)^3+(1+r\sin\theta)^3-3(1+r\cos\theta)(1+r\sin\theta)+1=0,$$

$$3r^2(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta)+r^3(\cos^3\theta+\sin^3\theta)=0.$$

Two values of r are zero, and the point (1, 1) is therefore a double point. But since no real value of θ will make another value of r zero, the point is a conjugate point.

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let us take the more general equation $x^3+y^3-3cxy+c^3=0$. Denoting this polynomial by F, we have

$$\frac{\partial F}{\partial x}=3x^2-3cy,\ \frac{\partial F}{\partial y}=3y^2-3cx.$$

Putting each $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$ equal to zero, we get $y=x=c$. It is easy to show that $H=\left(\frac{\partial^2F}{\partial x\partial y}\right)^2-\frac{\partial^2F}{\partial x^2}\frac{\partial^2F}{\partial y^2}=-27c^2$, being negative. Hence (c, c) is a conjugate point.